Signals & Systems

for

EC / EE / IN

By

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Syllabus for Signals & Systems

Continuous-Time Signals: Fourier Series and Fourier Transform Representations, Sampling Theorem and Applications; Discrete-Time Signals: Discrete-Time Fourier Transform (DTFT), DFT, FFT, Z-Transform, Interpolation of Discrete-Time Signals; LTI Systems: Definition and Properties, Causality, Stability, Impulse Response, Convolution, Poles and Zeros, Parallel and Cascade Structure, Frequency Response, Group Delay, Phase Delay, Digital Filter Design Techniques.

Previous Year GATE Papers and Analysis

GATE Papers with answer key

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Subject wise Weightage Analysis

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Contents

“Obstacles are those frightful things you can see when you take your eyes off your goal."
...Henry Ford

CHAPTER 1

Introduction to Signals & Systems

Learning Objectives

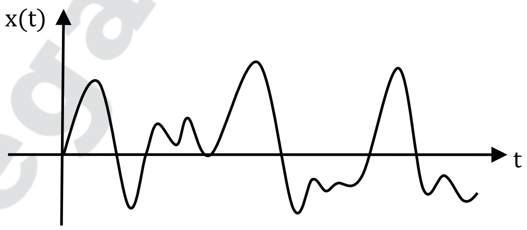
After reading this chapter, you will know:

1. Classifications of Signals
2. Basic Operations on Signals
3. Elementary Signals
4. Exponential
5. Characteristics of Systems

Introduction

Signal: Signal is defined as a function that conveys useful information about the state or behaviour of a physical phenomenon. Signal is typically the variation with respect to an independent quantity like time or distance as shown in figure below. Time is assumed as independent variable for remaining part of the discussion, unless mentioned.

(1) Speech signal – plot of amplitude with respect to time [x(t)]
(2) Image –plot of intensity with respect to spatial co-ordinates [I(x, y)]
(3) Video – plot of intensity with respect to spatial co-ordinates and time [V(x, y, t)]



Continuous –Time Signal

System: System is defined as an entity which extracts useful information from the signal or processes the signal as per a specific function.
E.g.: Speech Signal Filtering

Classification of Signals

Depending on property under consideration, signals can be classified in the following ways.

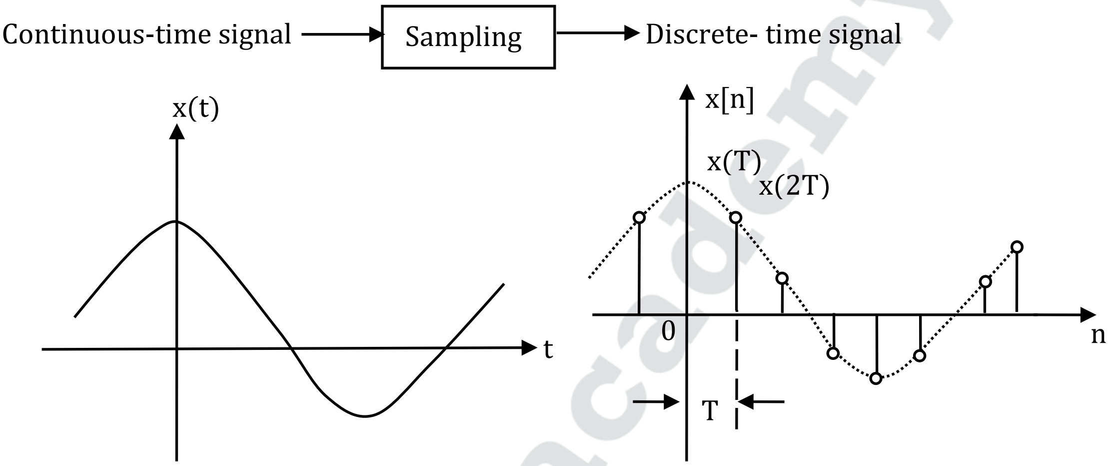
Deterministic vs Random Signals: A signal is said to be deterministic signal whose values can be predicted in advance.
E.g.: A sinωt

A signal is said to be random signal whose values are can't be predicted in advance.
E.g.: Noise

Continuous-Time vs Discrete-Time Signals: Continuous-time signal is defined as a signal which is defined for all instants of time. For example x(t)= at, where a is any constant . Discrete time signal is a signal which is defined at specific instants of time only and is obtained by sampling a Continuous – time signal. Hence, discrete-time signal is not defined for non-integer instants and is often identified as sequence of numbers, denoted as x [n] where n is integer.

$x[n] = x(t)|_{t=nT}\ \forall n = 0, 1, 2, 3....$

$x[n] = \{ x(0), x(T), x(2T)...... \}$



Demonstration of Sampling

In the figure shown above, x[n] is the discrete time signal obtained by uniform sampling of x(t) with a sampling period T.

Analog Signal vs Digital Signal: An analog signal is defined as a signal which can take any value between $-\infty$ to ∞ and can be defined at either specific or ray long instant of time
E.g.: $x(t) = \sin(at + \phi)$

Digital signal is defined as a signal which is defined at specific instants of time and also dependent variables can take only specific values. Digital signal is obtained from discrete-time signal by quantization.

Periodic vs Non-Periodic Signals: A continuous –time signal is periodic if there exists T such that
$x(t + T) = x(t),\quad \forall t ; T \in R - \{0\}$
If there is no such T then the signal is called non-periodic signal.
Example of periodic signal is sine waveform like x (t) = sin (t) and non-periodic is $x(t) = e^t$
Note: Even a non – periodic can be called periodic, with period ∞.

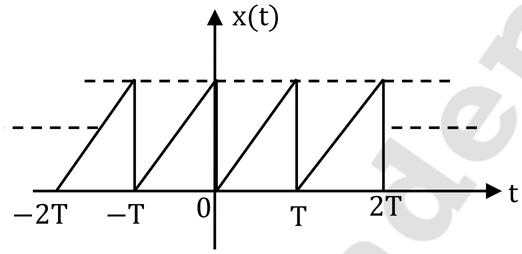
The smallest positive value of T that satisfies above condition is called fundamental period of x(t). Also, angular frequency of continuous-time signals is defined as, $\Omega = 2\pi/T$ and is measured in rad/sec.A discrete-time signal is periodic if there exists N such that
$x[n] = x[n + N], \forall n ;\ N \in Z - \{0\}$

The smallest positive N that satisfies above condition is called fundamental period of x[n]. Here N is always positive integer and angular frequency is defined as $\omega = 2\pi/N$ and is measured in radians/samples.

Note 1: If $x_1(t)$ and $x_2(t)$ are periodic signals with periods T_1 and T_2 respectively, then $x(t) = x_1(t) + x_2(t)$ is periodic if (if and only if) T_1/T_2 is a rational number and period of x(t) is least common multiple (LCM) of T_1 and T_2.

Note 2: If $x_1[n]$ is periodic with fundamental period N and $x_2[n]$ is periodic with fundamental period M then $x[n] = x_1[n] + x_2[n]$ is always periodic with fundamental period equal to the least common multiple (LCM) of M and N.

Figure below shows a signal, x(t) of period T



Example of a Periodic Signal

Real vs Complex Signals: A signal x(t) is real signal if its value are only real numbers and the signal x(t) is complex signal if its value are complex numbers.

Even and Odd Signals: A CTS x(t) is called even if $x(-t)= x(t)$ example $x(t) = \cos t$ and it is called odd if $x(-t) = -x(t)$

E.g.: $x(t)= \sin(t)$

Conjugate Symmetric vs Skew Symmetric Signals: A continuous time signal x(t) is conjugate symmetric if $x(t) = x^*(-t);\forall t$. Also, x(t) is conjugate skew symmetric if $x(t) = -x^*(-t);\ \forall t$.

Note: Any arbitrary signal x(t) can be considered to constitute 2 parts as below,

$x(t) = x_e(t)+ x_0(t)$

Where, $x_e(t) =$ conjugate symmetric part of signal $= \dfrac{\left(x(t) + x^*(-t)\right)}{2} = x_e{}^*(-t)$ and

$x_0(t) =$ conjugate skew symmetric part of signal $= \dfrac{\left(x(t) - x^*(-t)\right)}{2} = -x_0{}^*(-t)$

If signal x(t) is real, x(t) constitutes even and odd parts.

$x(t) = x_e(t) + x_0(t)$

Where, $x_e(t) = \dfrac{\left(x(t) + x(-t)\right)}{2}$ and $x_e(t) = x_e(-t)$

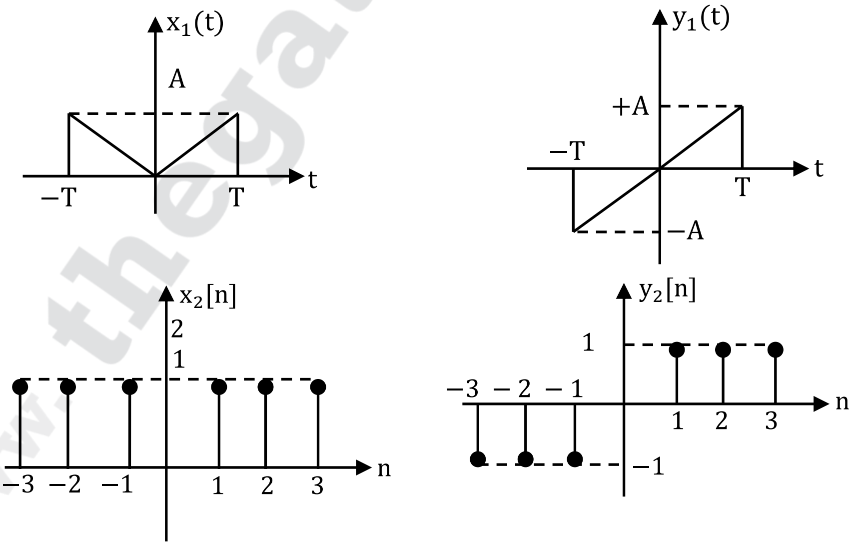
$x_0(t) = \dfrac{\left(x(t) - x(-t)\right)}{2}$ and $x_0(t) = -x_0(-t)$

Above properties can also be applied for discrete time signals and are summarized in the following table.

Symmetry Properties Based on Nature of Signal			
SL No.	Nature of Signal	Property	Condition
1.	Complex, continuous−time	Conjugate symmetry	$x(t) = x^*(-t)$
2.	Complex, continuous−time	Conjugate skew symmetry	$x(t) = -x^*(-t)$
3.	Real, continuous −time	Even function	$x(t) = x(-t)$
4.	Real, continuous −time	Odd function	$x(t) = -x(-t)$
5.	Complex, discrete −time	Conjugate symmetry	$x[n] = x^*[-n]$
6.	Complex, discrete−time	Conjugate skew symmetry	$x[n] = -x^*[-n]$
7.	Real, discrete−time	Even function	$x[n] = x[-n]$
8.	Real, discrete−time	Odd function	$x[n] = -x[-n]$

Decomposition Based on Nature of Signal			
SL No.	Nature of Signal	Decomposition	Properties
1.	Complex, continuous –time	$x(t) = x_e(t) + x_0(t)$	$x_e(t) = x_e^*(-t)$ $x_0(t) = -x_0^*(-t)$
2.	Real, continuous time	$x(t) = x_e(t) + x_0(t)$	$x_e(t) = x_e(-t)$ $x_0(t) = -x_0(-t)$
3.	Complex, discrete−time	$x[n] = x_e[n] + x_0[n]$	$x_e[n] = x_e^*[-n]$ $x_0[n] = -x_0^*[-n]$
4.	Real, discrete−time	$x[n] = x_e[n] + x_0[n]$	$x_e[n] = x_e[-n]$ $x_0[n] = -x_0[-n]$

In the figure shown below, $x_1(t)$, $x_2[n]$ are even signals and $y_1(t)$, $y_2[n]$ are odd signals.



Examples of Even and Odd Signals

If $x_{e_1}(t)$ and $x_{e_2}(t)$ are complex conjugate symmetric signals and $x_{0_1}(t)$ and $x_{0_2}(t)$ are complex conjugate skew – symmetric signals, then

(a) $x_{e_1}(t) + x_{e_2}(t)$ is conjugate symmetric

(b) $x_{0_1}(t) + x_{0_2}(t)$ is conjugate skew symmetric
(c) $x_{e_1}(t)\, x_{e_2}(t)$ is conjugate symmetric
(d) $x_{0_1}(t)\, x_{0_2}(t)$ is conjugate symmetric

Above applies for complex discrete-time signals also and can be equivalently derived for real signals, based on even-odd function properties.

Energy & Power Signals

The formulas for calculation of Energy, E and power, P of a continuous/discrete-time signal are given in table below,

Formulas for Calculation of Energy and Power		
SL No.	**Nature of the Signal**	**Formulas for Energy & Power Calculation**
1.	Continuous-time, non-periodic	$E = \int_{-\infty}^{+\infty} [x(t)]^2 dt\,;\, P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} [x(t)]^2 dt$
2.	Continuous-time, periodic signal with period T	$E = \int_{-\infty}^{+\infty} [x(t)]^2 dt\,;\, P = \frac{1}{T} \int_{-T/2}^{+T/2} [x(t)]^2 dt$
3.	Discrete-time, non-periodic	$E = \lim_{N\to\infty} \sum_{n=-N}^{N} \lvert x[n]\rvert^2\,;\, P = \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} \lvert x[n]\rvert^2$
4.	Discrete-time, periodic signal with period N	$E = \lim_{N\to\infty} \sum_{n=-N}^{N} \lvert x[n]\rvert^2\,;\, P = \frac{1}{(2N+1)} \sum_{n=-N}^{N} \lvert x[n]\rvert^2$

A signal is called energy signal if $0 < E < \infty$ and $P = 0$
A signal is called power signal if $0 < P < \infty$ and $E \to \infty$

Note: (1) Energy signal has zero average power.
(2) Power signal has infinite energy.
(3) Usually periodic signals and random signals are power signals.
(4) Usually deterministic and non-periodic signals are energy signals.

Basic Operations on Signals

Depending on nature of operation, different basic operations can be applied on dependent and independent variables of a signal. The table below basic summaries operations that can be performed on dependent variable of a signal.

Summary of Basic Operations on Dependent Variable of a Signal			
SL No.	**Operation**	**Continuous-Time Signal**	**Discrete –Time Signal**
1.	Amplitude scaling	$y(t) = c\,x(t)$	$y[n] = c\,x[n]$
2.	Addition	$y(t) = x_1(t) + x_2(t)$	$y[n] = x_1[n] + x_2[n]$
3.	Multiplication	$y(t) = x_1(t)\, x_2(t)$	$y[n] = x_1[n] x_2[n]$
4.	Differentiation	$y(t) = \frac{d}{dt}(x(t))$	$y[n] = x[n] - x[n-1]$

5.	Integration	$y(t) = \int_{T_1}^{T_2} x(t)dt$	$y[n] = \sum_{n=N_1}^{N_2} x[n]$

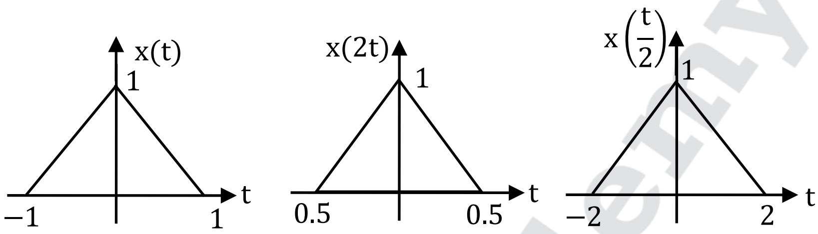
Similarly, following operations can be performed on independent variable of a signal.

Time Scaling

For continuous –time signals, $y(t) = x(at)$. If $a > 1$, $y(t)$ is obtained by compressing signal $x(t)$ along time axis by 'a' and vice versa.

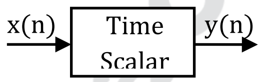
If $a < 1$, $y(t)$ is obtained by expanding signal $x(t)$ along time axis by 'a'

E.g.:



Demonstration of Time Scaling

In Discrete-time (D-T) signals, time scaling can be divided in to two ports. One is Decimation and the other is interpolation.



For a D-T signal $x(n)$, the scaled version of it is given by $y(n)$, where
$y(n) = x[kn]$ and k is scaling factor

(i) **Decimation:** If value of $k > 1$, it would lead to reduction of samples from the original D-T sequence. This process is known as Decimation (or) Down sampling.

Example: For a D-T sequence $x(n) = \{4, 3, 2, \underset{\uparrow}{1}, 2, 3, 4\}$, $x(2n)$, $x(3n)$ etc are the decimated sequences of $x(n)$ $x[2n] = \{3, \underset{\uparrow}{1}, 3\}$ and $x(3n) = \{4, \underset{\uparrow}{1}, 4\}$

From the above example it is clear that $x(2n)$ and $x(3n)$ are decimated sequences formed by selecting every second and every third sample respectively of $x(n)$ starting from $n = 0$

Note: In the process of decimation, care has to be taken that sequence is not affected by Aliasing.

(ii) **Interpolation**: If the value of $k < 1$, it would lead to increase in samples when compared to original sequence. This process is known as interpolation (or) up sampling.

Example: For the sequence $x(n)$ given in last example, find $x[n/2]$

$x[n/2] = \{4, \_, 3, \_, 2, \underset{\uparrow}{\_}, 1, \_, 2, \_, 3, \_, 4\}$

If the extra samples are added by zeros, it is known as zero interpolation.